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Simulation of passively Q-switched rate equation using saturable crystal Dy $^{+2}$: CaF_2 with ruby laser

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Abstract: The simulation of passively Q-switching is four non – linear first order differential equations. The optimization of passively Q-switching simulation was carried out using the constrained Rosenbrock technique. The maximization option in this technique was utilized to the fourth equation as an objective function; the parameters, γ_a, γ_c and β as were dealt with as decision variables. A FORTRAN program was written to determine the optimum values of the decision variables through the simulation of the four coupled equations, for ruby laser Q–switched by Dy $^{+2}$: CaF_2.For different Dy $^{+2}$:CaF_2 molecules number, the values of decision variables was predicted using our written program. The relaxation time of Dy $^{+2}$: CaF_2, used with ruby was calculated using the predicted value of γ_a.

Keywords: Passive Q-switching, rate equations, ruby laser, Rosenbrock optimization method, Dy $^{+2}$:CaF_2 crystal.

Introduction

Q-switched operation is very common and important method to provide short laser pulses with high peak power. Thus it has many applications in the fields of non-linear optics, range finding, medicine, and micromachining. Passive Q-switching, without extra power supply and driving, has attracted much research attention in recent years [1]. Performance optimization of Q-switched lasers also has received much attention for several work groups [2,3]. In general, there are two approaches to the optimization of key parameters such as the output coupler and initial saturable absorber transmissions.

The first method considers the rate equations describing the Q-switched laser, which result in the transcendental analytical relations between the chosen input parameters and the maximal output pulse energy and minimal pulse width. The obstacle here is the need for the additional recalculations to extract values of the optimal input parameters in the explicit form, which is appropriate for practical applications. The second method is creation of a family of graphical curves reflecting the laser output characteristics versus such input parameters as output coupler and Saturable absorber transmission.

The best (optimal) value of the input parameter is determined by the visual choice between curves with a maximal output energy or minimal pulse width. In this way, for an exact estimation of the optimum, it is necessary to go through the set of different curves. Degnan [4]

obtained the key parameters of energy – maximized passively Q-switched laser as function of two variables and generated several design curves. However, the excited state absorption of the saturable absorber was not considered in Degnans rate equations. Kuo and Brinbaum [5] studied the characteristics of ruby with Dy^{+2}:CaF_2 Q-switched laser theoretically and experimentally. The excited state absorption of the saturable absorber was considered in there rate equations. However, they did not deal with the optimization of the laser.

In this paper, the four coupled rate equations were solved, the operation of lasers passively Q-switched by saturable absorber with long excited state lifetime and appreciable excited state absorption was described. That is to obtain the expressions of pulse characteristics such as output energy, peak power, and pulse width. We then determine the parameters of an optimally coupled passively Q-switched laser as function of three variables.

The Models of Dy^{+2}: CaF_2 saturable absorber

Many researches represent a theoretical description of the dye saturable absorption had been conducted according to the number of energy levels that contribute to the absorption dynamics, two, three, four or multi-levels. The situation where a saturable absorber is characterized by only excited state seldom occurs in nature. In reality, a saturable absorber is usually characterized by more than one excited state to which molecules from ground state are transported. The molecules, which are located in excited state, can be transported even to the higher excited levels (on the condition that the energy gap of adjacent excited levels equals the energy of absorption quantum) [6].

The absorption dynamics may be described by a four – level system, as illustrated in Fig. (1). The incident photons excite molecules from the ground state (0) to an excited state (3). The excited molecules relax very fast to a temporal equilibrium position (2) of the excited state (τ_{32} very short) . From level (1) Molecules relax to the ground state (0).

Thereby residual absorption in the Saturable absorber interpreted as excited state absorption with a cross – section ($\sigma_{e.s.a}$)[5] .

The relaxation from level (3) to level (1) is generally very short. Saturable absorbers are characterized by (ground state absorption cross section $\sigma_{g.s.a} > \sigma_{e.s.a}$). Accordingly the bleaching of this system will require interval greater than that for both two and three – level system. Also the ground and the excited states define the absorption mechanism for this system [6].

Consequently, a four – level Saturable absorber representation is more actual than that of both two and three – level.

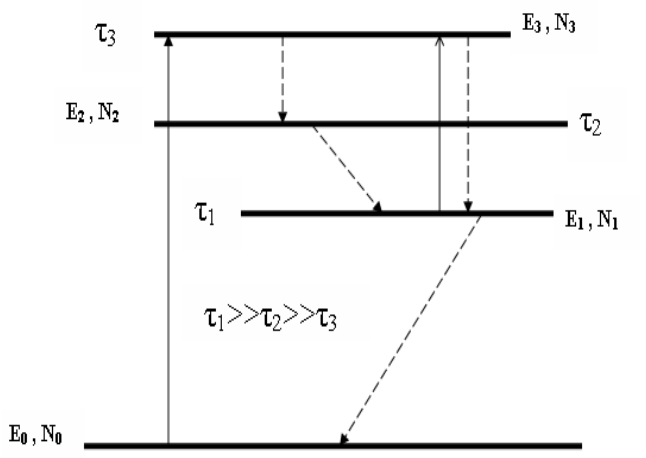

Fig. (1) Transition mechanisms of a four – level system ($E_2 \longrightarrow E_1$ is the laser action)[7].

The important difference that distinguish the four - level model with the two and three levels models are:

1. Transfer of molecules is not stopped when the populations of upper and lower levels of the saturable absorber are roughly equal [8].
2. The total absorption mechanism concerns both the ground and first excited state absorption with a cross-section ($\sigma_{g.s.a}$) and ($\sigma_{e.s.a}$) respectively.
3. The ratio of the first excited state absorption cross-section ($\sigma_{e.s.a}$) to ground state absorption cross-section ($\sigma_{g.s.a}$) does not equal to zero, it is equal to a constant value , *that is:* $\beta=(\sigma_{e.s.a}/\sigma_{g.s.a})$.

This model takes into account residual absorption in the dye resulting from excited state absorption with cross-section ($\sigma_{e.s.a}$).

Rate equations

Four coupled rate equations models were used to describe the rate equations for passively Q-switched laser accurately according to the absorption mechanism of the saturable absorber represented in this system [9].

In this model the contribution of the first absorption excitation state absorption of the stimulated photons is taken into account. Consequently, this leads to the decrease the rate of the cavity photon number; the equations are [10]:

$$\frac{dn}{dt} = [K_g N_g - K_a N_a - \beta K_a N_{au} - \gamma_c]n \quad (1)$$

$$\frac{dN_g}{dt} = R_p - \gamma_g N_g - 2 * K_g N_g n \quad (2)$$

$$\frac{dN_a}{dt} = \gamma_a N_{au} - K_a N_a n \quad (3)$$

$$\frac{dN_{au}}{dt} = K_a N_a n - \gamma_a N_{au} \quad (4)$$

where,

n: The photon number inside the laser resonator.
N_g: The population inversion; i.e the molecule number accumulated at the upper laser level.
N_a: The ground state population of the saturable absorber.
γ_g = $1/\tau_g$ is the effective decay rate of the upper laser level(sec^{-1}) .
τ_g: The upper laser level lifetime (sec).
R_p : The pumping rate (sec^{-1}).
K_g: The coupling coefficient between the stimulated photons and the active medium element [5] .

Thus K_g appears in the term which represents the stimulated emission. Also, it refers to the probability of the stimulated emission per unit time; its value can be calculated using K_a which is the coupling coefficient between the stimulated photons and the saturable absorber molecules [5]. Thus K_a appears in the term which represents absorbance of the stimulated photons .Also, it refers to the probability of absorption of the stimulated photons per unit time.
The cavity decay rate is given by:

$$\gamma_c = \frac{1}{\tau_c}$$

Where τ_c is the photon lifetime inside the cavity.

β is the ratio of the excited state absorption cross-section ($\sigma_{e.s.a}$**)**, to the ground state absorption cross-section ($\sigma_{g.s.a}$**)**, of the saturable absorber molecules.

N_{au} is the first excited state population of the saturable absorber.
γ_a is saturable absorber decay rate(sec^{-1}).

Constrained RosenBrock optimization method

This method deals with the objective function while minimizing or maximizing that function. It also deals with constrained decision parameters [11]. In our problem, the fourth equation in the rate equations, equation (4), was used as an objective function. This equation is a coupling one of the three equations (1, 2, & 3), for computing n, Ng, Na which are used in evaluating Nau. Constrained limits for β, as decision parameters of (0.2-0.9) was used.

Results and Discussion

Optimum Values of β at different Values of initial Saturable Absorber Molecules Number

The saturable absorber number of Dy^{2+}: CaF_2 molecules (ni) has been increased in the range of (5.18 – 12) $\times 10^{15}$ molecules. The results for prediction three decision variables (γ_a , γ_c and β), and two decision variables (γ_a, β) and (γ_c, β) are listed in tables (1), (2) and (3) respectively.

All the obtained results are predicted at the maximization of the objective function N_{au} which is the saturable absorber molecules number at first excited state.

In the tables, the values of the objective function are approaching the initial values of initial molecules number, because at bleaching point, almost the saturable molecules absorber will be transferred from the ground state to the first excited state.

From the results one can follow the increase. As the number of molecular increases, $\sigma_{e.s}$ will be increased by a non – significant amount at a certain value of molecular number; which will not affect the value of β. At the same time σ_g will not be increased.

This is because of the increase in the absorption cross section of saturable absorber at the first excited state. Figure (2) shows β as a function of saturable absorber molecules number (ni).

Table (1): Optimum values of β, γ_a, γ_c at different Molecules Numbers.

ni mole.×10^{15}	Optimum β value	Optimum γa value Sec $^{-1}$	Optimum γc value Sec $^{-1}$	Period nsec	Objective Function ×10^{15} mole.
	0.2035	6666.4	3.42000000×10 ^{8}	26.2	5.1771
6	0.2555	7179.29	3.42000000×10 ^{8}	22.45	5.9968
7	0.385	8070.46	3.42717445×10 ^{8}	20.22	6.9966
8	0.3480	8279.58	3.40060019×10 ^{8}	19.8	7.9964
9	0.3995	8792.83	3.40000000×10 ^{8}	19.41	8.9961
10	0.4523	8798.41	3.400088572×10 ^{8}	19	9.9957
11	0.4903	8801.65	3.421523359×10 ^{8}	18.69	10.995
12	0.5080	8881.62	3.421523359×10 ^{8}	18.45	11.995

The values of β in Tables (1), (2) and (3) are nearly the same that indicates the correct selection of the optimization manner in the predictions of these parameters.

Optimum Values of Average Cavity Photon decay Rate at different values of initial Saturable Absorber Molecules numbers

To choose the optimum value for γ_c which is the fourth term in equation (4), the maximization of n_{au} (saturable absorber molecules number at 1stexcited state) was used. That is because of the saturable absorber behavior; at bleaching, the most absorber molecules are transferred to the first excited state.

The values of γ_c are listed in tables (1) and (2). The values of γ_c are very close to each others. That is because of the independency of the cavity photon lifetime upon the number of saturable absorber; as the nonlinear behavior of the saturable absorber.

Table (2): Optimum values of β, γ_c at different Molecules Numbers.

ni mole.×10^{15}	Optimum β value	Optimum γa value Sec $^{-1}$	Period nsec	Objective Function ×10^{15} mole
5.18	0.2053	3.42×10 ^{8}	26.35	5.1771
6	0.2303	3.39002×10 ^{8}	22.3	5.9968
7	0.2774	3.30853×10 ^{8}	20	6.9966
8	0.3480	3.400001×10 ^{8}	19.8	7.9964
9	0.3995	3.4×10 ^{8}	19.5	8.9961
10	0.4523	3.40009×10 ^{8}	19	9.9957
11	0.5006	3.40717×10 ^{8}	18.9	10.995
12	0.5150	3.42152×10 ^{8}	18.29	11.995

Optimum Values of Average Saturable Absorber Decay Rate at different initial Values of Molecules number of Saturable Absorber

The optimum value of γ_a at each case was predicted according to the maximization criteria.

These values are listed in Tables (1) and (3) One can observe the increase in γ_a values as increasing the saturable absorber molecules number (ni) till reaching certain value of (ni) (8×10^{15}).

The variation is very small and one can say that it is fixed at this value.

Table (3): Optimum values of β, γ_a at different Molecules Numbers

ni $\times 10^{15}$ mole.	Optimum β value	Optimum γ_a value Sec $^{-1}$	Relaxation Time nsec.	Period nsec.	Objective Function $\times 10^{15}$ mole.
5.18	0.2078	6666.4	1.5×10^5	25.9	5.1771
6	0.2578	7179.29	1.392895444×10^5	22.1	5.9968
7	0.385	8070.46	1.239087×10^5	20.2	6.9966
8	0.3880	8279.58	1.207791×10^5	19.7	7.9964
9	0.3995	8792.83	1.13729×10^5	19.4	8.9961
10	0.4736	8798.41	1.136568×10^5	18.9	9.9957
11	0.4903	8801.65	1.136151×10^5	18.5	10.995
12	0.5080	8881.62	1.12592×10^5	18.4	11.995

That is because; the number of molecules at each energy level is limited according to Boltzman distribution [12]. And the increase of the number of Dy^{2+}: CaF_2 molecules at the ground state will cause slightly increase in the number of molecules at the first excited level. Subsequently that will not increase the inter system crossing rate, which is the affecting parameter in reducing the relaxation time and so the recovery time which is related to the pulse duration. Table (3) shows also the values of relaxation time that was calculated using the predicted γ_a. Fig (3) shows the profiles of relaxation time and pulse duration as a function of Dy^{+2}: CaF_2 molecules numbers .

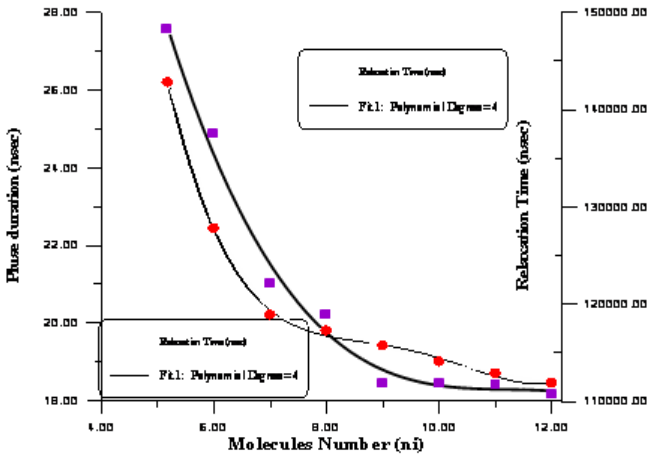

Fig. (3): The profile of relaxation time ■ and pulse duration● as a function of Dy ^{2}: CaF $_{2}$ molecules numbers.

Stimulated Photon cavity Profiles

The giant laser pulse profiles are shown in Fig.(4) while using the predicted values (β, γ_c) of the rate equations parameters. These profiles; are of three cases, they are obtained at saturable absorber molecules numbers of (5.18, 9 and 12) $\times 10^{15}$ molecules.

The maximum number of the stimulated photons for each case is related to the initial saturable absorber molecules number. At 12×10^{15} molecules, the time for bleaching the saturable absorber is the largest for the three cases. Consequently, the maximum number of stimulated photons is maximum for the three cases.

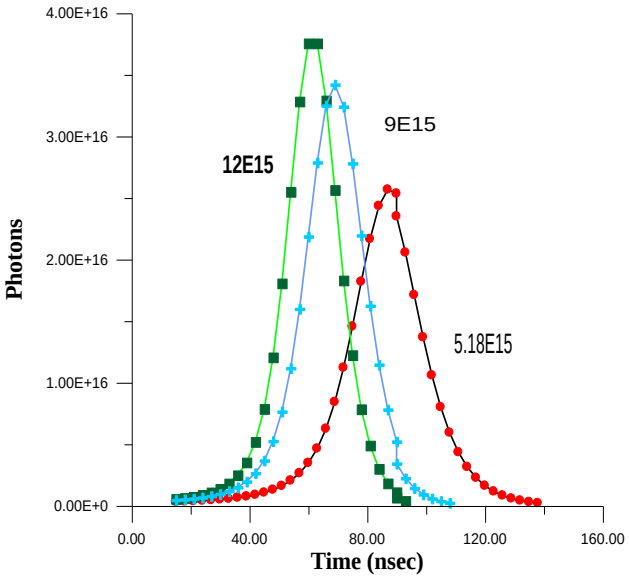

Fig. (4): The stimulated photon profiles.

Conclusions

Constraint Rosenbrock optimization technique is suitable for dealing with the simulation passively Q- switching laser rate

equations. Rosenbrock optimization technique will be more reliable while dealing with many decision variables. While increasing the saturable absorber molecules numbers, R_T, τ_p are decreasing and γ_a, β are increasing till reaching certain values at a certain value of (ni), they will be constant.

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محاكاة معادلات المعدل باستخدام البلورات المشبعة مع ليزر الياقوت

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الخلاصة

في هذا البحث درست محاكاة نبضة الخرج الليزري بالنمط المفتاحي السلبي لمعادلات المعدل وذلك باستخدام تقانة امثلية روزنبرك المقيدة تم كتابة برنامج بلغة فورتران لتحديد امثل قيم لمتغيرات القرار من خلال الحل العددي لمعادلات المعدل انيا لمنظومة ليزر الياقوت التي تستخدم $Dy^{-2}:CaF_2$ كمفتاح عامل النوعية . قيم $\beta, \gamma_c, \gamma_a$ تم التنبؤ بها لعدة قيم من عدد جزيئات المادة الماصة المشبعة $Dy^{-2}:CaF_2$ زمن الاسترخاء للمادة الماصة المشبعة حسب من خلال قيم γ_a المستخرجة من البرنامج .